Interest rate sensitivities under the G2++ model

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The two-factor Hull-White (2-HW) model is a famous stochastic model that describes the instantaneous short rate. It has functional qualities required in various practical purposes as in Asset Liability Management and in Trading of interest rate derivatives. The 2-HW is actually a special case of a two-additive-factor Gaussian model G2++, which despite of its unpleasant feature of theoretical possibility of negative rates, is quite analytically tractable in that explicit formulas for basic instruments can be readily available.

Though closed-formulas for the prices of various main interest-rate instruments are known and used under the G2++ model, it seems that references for the corresponding sensitivities are not clearly presented over the financial literature. To fill this gap is among of our purposes in the present work.

So we derive here analytic expressions for the sensitivities of zero-coupon bond, coupon-bearing bonds, forward rate agreement and interest rate swap contracts. The sensitivities under consideration here are those with respect to the shocks linked to the unobservable two-uncertainty shocks risk/opportunity factors underlying the G2++ model. As a such, the hedging of a position sensitive to the interest rate by means of a portfolio (in accordance with the market participants practice) becomes easily transparent as just resulting from the balance between the various involved sensitivities. The systematic analysis of using of a portfolio as hedging instrument as done in this paper seems to be new, since very often authors are essentially focused on the use of one type of instrument.

The idea of hedging is to compensate the adverse eect of the position to hedge by the hedging instrument cash-ow. That is, if the position to cover losses then one expects to get sucient gains from the hedging instrument. The operation is not intended to get any prot, but to try to maintain the position in its level at the hedge starting.

In the technical aspect, the idea in hedging is that the change of each position can be represented by some polynomial function whose the underlying variable is the associated shock.

This results an optimization problem in integer numbers, non linear objective function, non convex domain, with linear constraint. It's an NP-hard problem. By using linearization technics, we are reduced to a Mixed Integer Linear Problem. This last may be solved using CPLEX commercial solver.

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A numerical applications are available in the final of the work to prove the importance of our hedging formulations and ideas.

Keywords: interest rate sensitivities, two-factor Hull-White model, G2++ model, hedging.

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