Tensor Krylov subspace methods via the T-product for large Sylvester tensor equations

Fatima BOUYGHF[1]

[1]*LMNO, University of Caen Normandie, France*
[1] *fatimabouyghf3@gmail.com*
Joint work with Alaa El ICHI from ULCO (France)
and Mohamed El GUIDE from UMIVP (Morocco)

The aim of this talk is to present numerical Tensor Krylov subspace methods for solving the Sylvester tensor equation

$$\mathcal{M}(\mathcal{X}) = \mathcal{C}, \tag{1}$$

where $\mathcal{M}$ is a linear operator that could be described as

$$\mathcal{M}(\mathcal{X}) = \mathcal{A} \star \mathcal{X} - \mathcal{X} \star \mathcal{B} \tag{2}$$

where $\mathcal{A}$, $\mathcal{X}$, $\mathcal{B}$ and $\mathcal{C}$ are three-way tensors leaving the specific dimensions to be defined later, and $\star$ is the T-product introduced by Kilmer and Martin [2].
Consider the following Sylvester matrix equation

$$AX + XB = C. \tag{3}$$

In the literature, several methods to solve equation (3) have been established. When matrices are of small sizes, the well-know direct methods are recommended. These methods are based on Schur decomposition to transform the original equation into a form that is easily solved by a forward substitution. For large Sylvester matrix equations, iterative projection methods have been developed, see for example [3, 4]. These methods use Galerkin projection methods, such the classical and the block Arnoldi techniques, to produce low-dimensional Sylvester matrix equations that are solved by using direct methods.
In the current talk, we are interested in developing robust and fast iterative Krylov subspace methods via T-product to solve the Sylvester tensor equation *STE* (1). In fact, when the tensors in equations (1) are of small sizes, the purpose is to extend matrix version of direct methods to third order tensors using the T-product formalism. This give us the *t-Bartels-Stewart* algorithm. For large size tensors, we describe first a new methods that will be defined as orthogonal and oblique projection onto a tensor Krylov subspace. In particular, the tensor Full orthogonalization method (tFOM) and tensor generalized minimal residual method (tGMRES) will be examined. We will also introduce the well-Know tensor Tubal block Krylov methods via T-product for transforming the original large Sylvester equation to a low-dimensional *STE* . In particular, we will describe the Tubal Block Arnoldi (TBA) as a generalization of the block Arnoldi matrix [5].

References

[1] F. Bouyghf, A. Messaoudi and H. Sadok, *A unified approach to Krylov subspace methods for solving linear systems,* Accepted for publication in Numerical Algorithms Journal.

[2] M. E. Kilmer and C. D. Martin, Factorization strategies for third-order tensors, Linear Algebra Appl., 435 (2011) 641-658.

[3] A. El Guennouni, K. Jbilou, A.J. Riquet, Block Krylov subspace methods for solving large Sylvester equations, Numer. Algebra 29 (2002) 75-96.

[4] D.Y. Hu, L. Reichel, Krylov subspace methods for the Sylvester equation, Linear Algebra Appl. 174 (1992) 283-314

[5] M. El Guide, A. El Ichi, K. Jbilou, and R. Sadaka, "On tensor GMRES and Golub-Kahan methods via the T-product for color image processing", Electronic Journal of Linear Algebra, 37(2021) 524-543.