

Analysis of an observers strategy for initial state reconstruction in unbounded domains.

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The general motivation of this work is the development of elastography techniques [1]. This medical imaging procedure aims to identify the physical properties of a biological tissue via the measurements of the wave propagation inside. More precisely, in this talk, we are interested in the problem of recovering a compactly supported initial state of the wave equation in an unbounded domain, given the velocity measurement in a bounded observation region surrounding the support of the initial state. We assume the domain of waves propagation to be unbounded to simulate the situation where the wavelength and the observation region are very small compared to the whole domain (as it occurs in elastography).

To solve this problem, we consider an iterative algorithm of reconstruction based on back and forth nudging [2] (also known as an observers strategy). Our goal is to analyze this algorithm (convergence and robustness with respect to noisy datas) and emphasizes the difficulties raised by considering an unbounded domain. In particular, to prove the convergence of the algorithm, we show a Geometrical Control Condition [3], [4] restricted to a class compactly supported initial data.

References

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